# Dynamic Systems Modeling Aids in the Design of Dispensing Systems

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**FOCUS:** Modeling complex mechanical systems can give engineers insight into the operation of these systems and can serve as the basis for design optimization. In this article a simulation technique known as Dynamic Systems Modeling is presented and applied to the design of a trigger spray dispenser. The methods described can be used to simulate a wide range of products and processes including liquids filling systems, packaging lines, and dispensing closures.

## Introduction

Dispensing systems for consumer products packaging are complex fluid-mechanical devices. These devices include multiple flow passages with tight tolerances and small cross-sectional areas, valves (both active and passive), and one or more pistons. These pumps must be capable of priming with as few strokes as possible, must work with products having a variety of physical properties, and must deliver consistent, reliable performance over the life of the package.

The ambitious design goals for dispensing pumps have traditionally been met by trial-and-error design methods. As the demands on these systems increase, these traditional methods are being supplemented through the use of computational simulation. Using simulation the designer can test a wide variety of design concepts before incurring the expense of prototyping. In addition, a well-designed simulation will allow the designer a "look inside" the dispenser that can guide design improvement in a rational fashion.

This paper discusses a method of simulating dispensing systems referred to as "Dynamic Systems Modeling." The methodology used in this approach is well established and has been applied to piping-system modeling, modeling of pumps and compressors, and the simulation of liquids filling lines. In the discussion below these methods are applied to a relatively simple and commonly available trigger spray in order to demonstrate the power of this technique in solving design problems in the packaging of consumer products.

## Dynamic Systems Model - "*TrigSpray*"

The process of developing and using a dynamic systems model will be illustrated by developing a simulation of the trigger sprayer shown in Figure 1. This sprayer uses a piston and three valves to allow priming, dispensing, and venting of the bottle. The ball check valve is the most obvious of the three valves and one of the more challenging features to simulate. The ball check

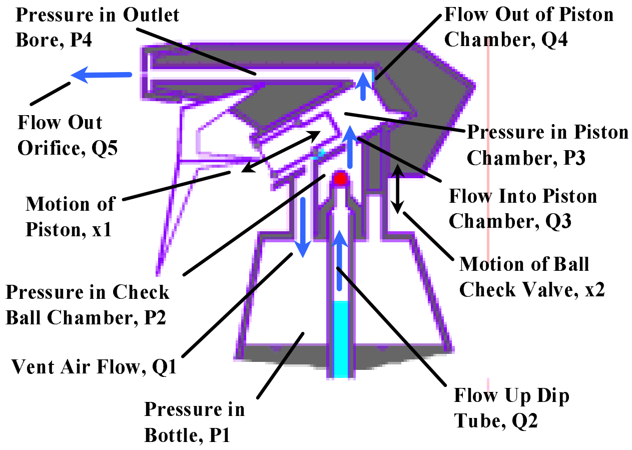


**Figure 1**
**Trigger Sprayer Geometry and Critical Variables**

valve prevents liquid from flowing back into the dip tube while the pump is dispensing. The performance of the ball check valve determines how many strokes are required to prime the pump since it both stops liquid from flowing back into the dip tube and also creates drag on the liquid flowing up into the pump. The second valve is in the spray nozzle itself. This valve works in opposition to the ball check valve, allowing flow during the dispensing portion of the stroke and closing during the suction portion of the stroke. This combination of opposed upstream and downstream check valves is typical of all piston-type pumps. The third valve in the system governs the venting of the package. This valve is built into the piston such that as the piston reaches the end of its stroke the outer piston seal is broken, opening an airflow path between the bottle's head space and the outside atmosphere. Because fluid has been withdrawn from the bottle during the suction stroke, the pressure in the bottle is negative when the vent is opened. It is important to relieve this negative pressure in order to preserve the shape of the package (to avoid paneling) and insure the consistent functioning of the pump.

The first step in developing a dynamic systems model is to identify the critical variables. These variables consist of flow rates between various chambers of the pump, pressures at various locations, and the motion of the mechanical components of the dispenser. These variables are identified for the trigger sprayer in Figures 1 and 2. Flow rates are denoted as Q, pressures as P, and mechanical displacements as x. In addition to the quantities shown in Figure 2, the simulation must keep track of the percentage of air in each chamber of the pump (represented by boxes in Figure 2).

Figure 2 illustrates that the trigger sprayer can be divided into three basic elements:

1) A pipe-like element describing flow between two chambers,

2) A volumetric element describing the pressure change within a chamber,

3) A mechanical element describing the motion of the check valve.

These elements are illustrated in Figures 3 – 5, respectively. Each of these elements provides an equation of change that describes how one or more of the variables shown in Figure 2 evolve with time.

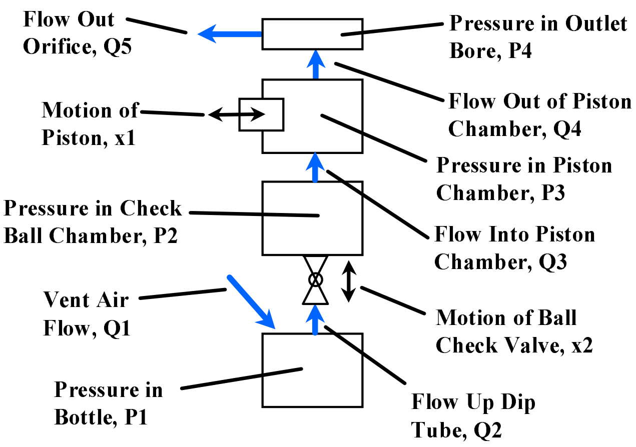


Figure 2
Block Diagram of Trigger Sprayer Model

The fluid flow element shown in Figure 3 is used to compute the change in the flow rate Q through the equation

$$dQ/dt = (A/L)\,(P_{in} - P_{out} - P_{loss} - P_{head})/\rho, \qquad (1)$$

where A is the cross-sectional area of the flow path, L is its length, $P_{loss}$ is the pressure loss due to friction and minor losses such as bends, $P_{head}$ is the change in head pressure between the upstream and downstream chambers, and ρ is the density of the fluid.

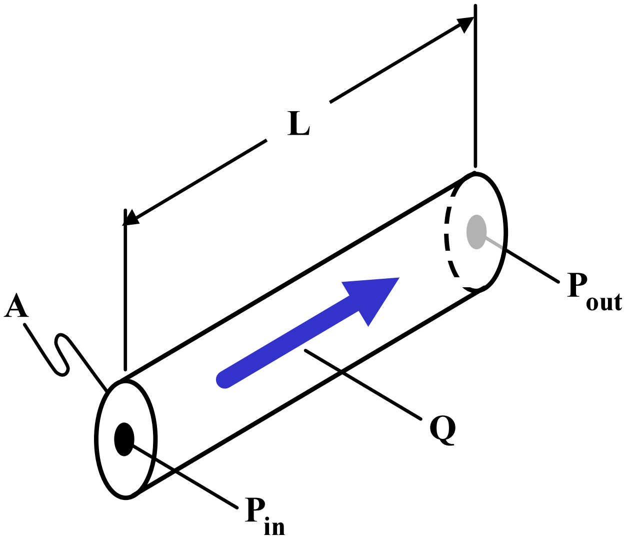


Figure 3
Fluid Flow Element

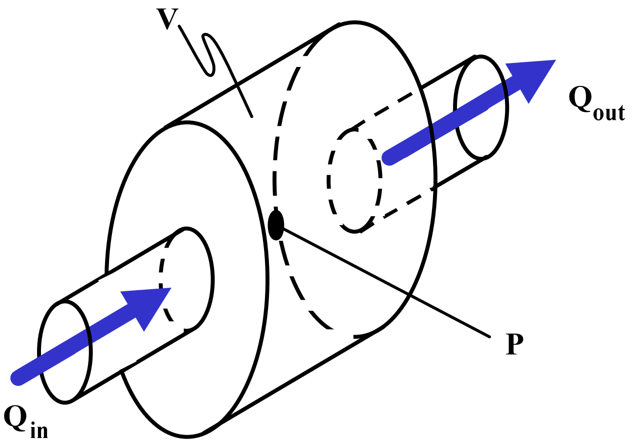


**Figure 4**
**Fluid Volume Element**

The fluid volume element shown in Figure 4 is used to compute the change in pressure P through the equation

$$dP/dt = (\beta/V)\,(Q_{in} - Q_{out}), \qquad (2)$$

where V is the volume of the chamber and β is the combined bulk modulus of the liquid and chamber. The bulk modulus of water is 316,000 psi. However, due to the presence of air bubbles and the stretching of the plastic walls of the sprayer when exposed to pressure, this value can drop as low as 20,000 psi. Equation (2) expresses the fundamental requirement that any mismatch between the flow entering a volume and the flow leaving that volume must be accounted for by expansion or compression of the fluid in the volume and an accompanying pressure change.

The final element is the check valve element, shown in Figure 5. This is frequently the most difficult feature to simulate. The equation for the motion of the check ball is taken directly from Newton's Second Law, giving

$$d^2x/dt^2 = (F_{drag} - W_{ball} - F_{seat}) / M_{ball}, \qquad (3)$$

where $F_{drag}$ is the drag on the ball check, $W_{ball}$ is the weight of the ball, and $F_{seat}$ is the reaction force on the seat when the ball is in its full open or full closed position.

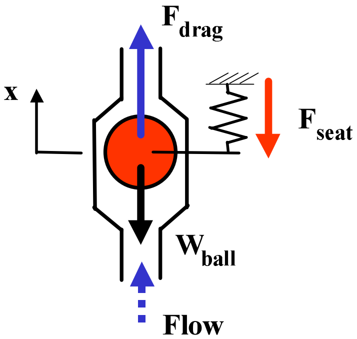
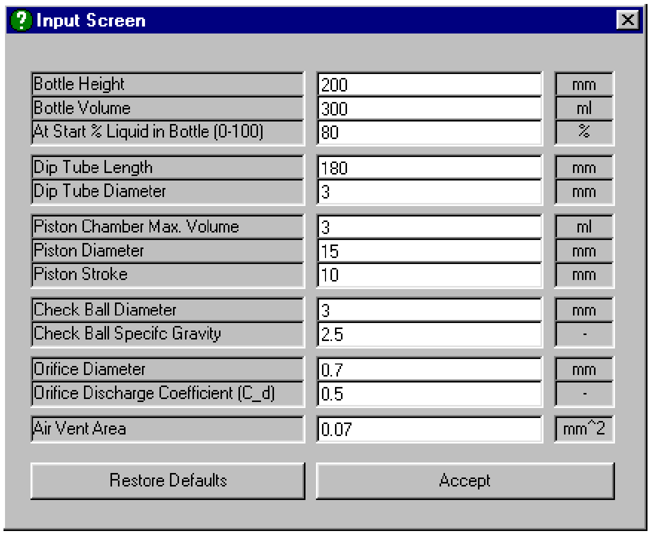


**Figure 5**
**Check Valve Element**

It is easy to see that a large, highly coupled set of equations results when equations (1) – (3) are applied to the calculating the evolution of Q1 – Q5, P1 – P4, and x2 shown in Figure 2. In addition, if air is present, as in the priming phase of operation, equations (1) – (3) must be adapted to include air as a second fluid, increasing the complexity of the equation set. Solution of these equations over the course of 10 seconds of simulated time will require evaluation of each equation more than 1 million times. Fortunately, the increasing speed of PC's has reduced the calculation time for this typical 10-second simulation to a reasonable 5 – 10 minutes. Therefore, a simulation of this type can readily be used by a design engineer to test a wide variety of design alternatives over the course of a single day. In the next section a design application of the resulting program, named *TrigSpray*, will be illustrated.

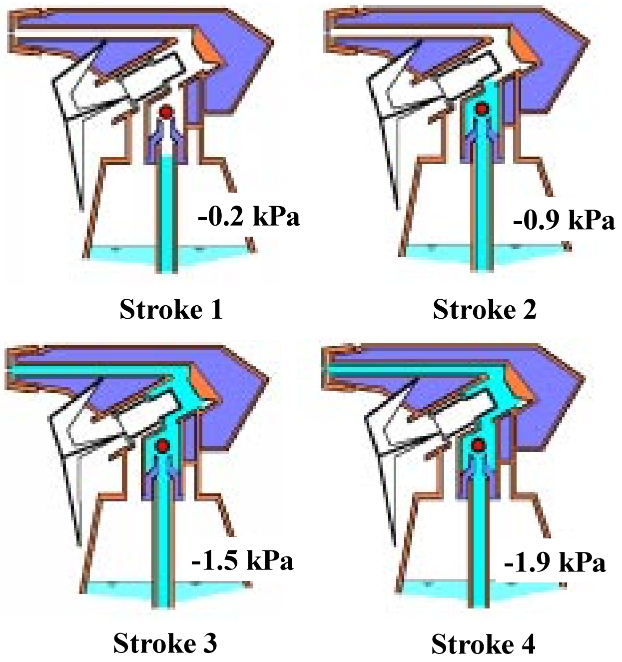
## Example – Predictive Design Using the *TrigSpray* Dynamic Systems Model

In this section the usefulness of the *TrigSpray* Dynamic Systems Model will be illustrated in a simple application, the sizing of the air vent through the outer piston seal. Figure 6 shows the input panel for the *TrigSpray*



| | | |
|---|---|---|
| Bottle Height | 200 | mm |
| Bottle Volume | 300 | ml |
| At Start % Liquid in Bottle (0-100) | 80 | % |
| Dip Tube Length | 180 | mm |
| Dip Tube Diameter | 3 | mm |
| Piston Chamber Max. Volume | 3 | ml |
| Piston Diameter | 15 | mm |
| Piston Stroke | 10 | mm |
| Check Ball Diameter | 3 | mm |
| Check Ball Specifc Gravity | 2.5 | - |
| Orifice Diameter | 0.7 | mm |
| Orifice Discharge Coefficient (C_d) | 0.5 | - |
| Air Vent Area | 0.07 | mm^2 |

**Figure 6**
**Dimensions and Property Inputs**



Stroke 1 Stroke 2

Stroke 3 Stroke 4

**Figure 7**
**Priming of Sprayer**

program. A number of user-defined inputs are available describing the geometry of the bottle and the design-critical components of the sprayer. At the bottom of the list is the air vent area. As a first guess we set this value at 0.07 mm$^2$ which corresponds to small but reasonable dimensions for an opening through the piston seal.

To test the proposed design, *TrigSpray* is run for five strokes over 10 seconds. Figure 7 shows the priming of the pump over the first four strokes and the pressure in the bottle just after closure of the air vent. This figure shows that the pump requires three strokes to prime, which is in agreement with tests on actual sprayers. The figure also shows that the pressure in the bottle decreases with each stroke. This effect is illustrated in more detail in Figure 8, which provides a graph of the piston motion and the pressure in the bottle over the first five strokes. The graph shows that during each stroke the bottle pressure decreases as liquid is drawn up through the dip tube, holds steady once the check valve is seated, and then recovers once the vent opens during the last 10% of the stroke. However, the pressure does not recover completely and over several strokes a significant vacuum begins to develop in the bottle. This vacuum will cause the bottle sides to collapse (panel), impairing the esthetics of the package.

The venting problem shown in Figures 7 and 8 can be easily corrected by increasing the area of the air vent. The magnitude of the required increase can be found by first doubling the vent area to 0.14 mm$^2$ and repeating the analysis. The results for this revised design are shown in Figure 9. Comparing this figure with Figure 8 shows that the bottle pressure profile for the new design is similar to that for the first design, with the exception that in the new design the bottle pressure recovers all the way back to zero at the end of each stroke. Therefore, doubling the size of the air vents meets the design goal for the pump. Additional iterations can be done to determine where the performance begins to degrade

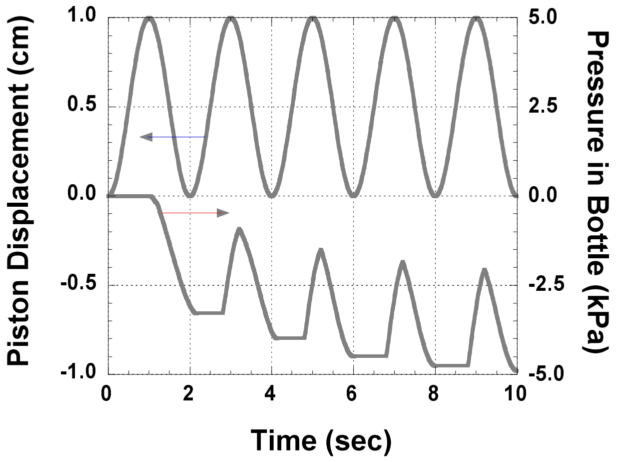


**Figure 8**
**Simulation Results for Original Design**

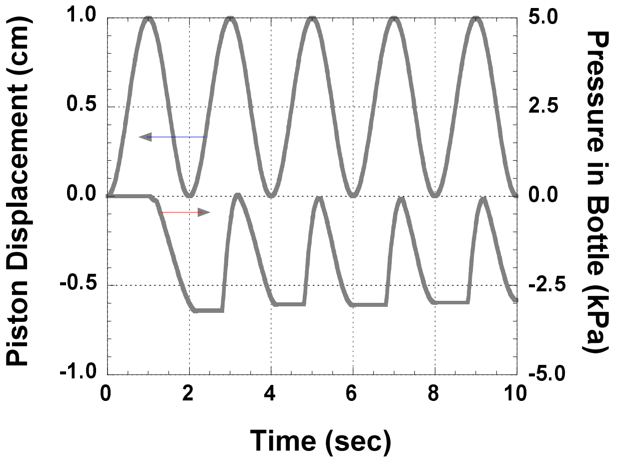


**Figure 9**
**Simulation Results for Improved Design**

when the vent area is incremented between 0.07 to 0.14 mm$^2$.

The *TrigSpray* program can be used to size various other components of the sprayer such as the piston and dip tube. In addition, the effect of fluid properties such as density and viscosity can be investigated so as to evaluate the functionality of proposed designs over a wide range of products.

## Conclusion

The model of the trigger sprayer described in this article provides a simple example of the power of dynamic systems modeling. The range of application of this technology is extremely broad. The author has used the methods illustrated here to simulate filling systems, part reject systems on production lines, aerosol cans, dryers, waterhammer in pipelines, and a wide variety of other fluid/mechanical systems. The important feature of all of these systems is that they involve dynamic events of sufficient complexity that hand calculations become too difficult to use for design. These systems also have characteristics such as mixed scales of large and small passages that make simulation using commercially available computational fluid dynamics (CFD) programs prohibitively expensive. For this class of problems a custom-written dynamic systems model provides a powerful desktop tool for the design engineer.